Asset Valuation

- Function of both return and risk
  - At the center of security analysis
- How should realized return and risk be measured?
  - The realized risk-return tradeoff is based on the past
  - The expected risk-return tradeoff is uncertain and may not occur

Return Components

- Returns consist of two elements:
  - Periodic cash flows such as interest or dividends (income return)
    - “Yield” measures relate income return to a price for the security
  - Price appreciation or depreciation (capital gain or loss)
    - The change in price of the asset
- Total Return = Yield + Price Change

Risk Sources

- Interest Rate Risk
  - Affects income return
- Market Risk
  - Overall market effects
- Inflation Risk
  - Purchasing power variability
- Business Risk
- Financial Risk
  - Tied to debt financing
- Liquidity Risk
  - Marketability with-out sale prices
- Exchange Rate Risk
- Country Risk
  - Political stability

Risk Types

- Two general types:
  - Systematic (general) risk
    - Pervasive, affecting all securities, cannot be avoided
    - Interest rate or market or inflation risks
  - Nonsystematic (specific) risk
    - Unique characteristics specific to issuer
- Total Risk = General Risk + Specific Risk
Measuring Returns

- For comparing performance over time or across different securities
- Total Return is a percentage relating all cash flows received during a given time period, denoted $CF_t + (PE - PB)$, to the start of period price, $PB$

$$TR = \frac{CF_t + (PE - PB)}{PB}$$

Measuring Returns

- Total Return can be either positive or negative
  - When cumulating or compounding, negative returns are problem
- A Return Relative solves the problem because it is always positive

$$RR = \frac{CF_t + PE}{PB} = 1 + TR$$

Measuring Returns

- To measure the level of wealth created by an investment rather than the change in wealth, need to cumulate returns over time
- Cumulative Wealth Index, $CWIn$, over $n$ periods, =

$$WI_0 (1 + TR_1)(1 + TR_2)...(1 + TR_n)$$

Measuring International Returns

- International returns include any realized exchange rate changes
  - If foreign currency depreciates, returns lower in domestic currency terms
- Total Return in domestic currency =

$$RR \times \frac{End \ Val. \ of \ For.Curr.}{Begin \ Val. \ of \ For.Curr.} - 1$$

Measures Describing a Return Series

- $TR$, $RR$, and $CWI$ are useful for a given, single time period
- What about summarizing returns over several time periods?
- Arithmetic mean, or simply mean,

$$\bar{X} = \frac{\sum X}{n}$$

Arithmetic Versus Geometric

- Arithmetic mean does not measure the compound growth rate over time
  - Does not capture the realized change in wealth over multiple periods
- Geometric mean reflects compound, cumulative returns over more than one period
### Geometric Mean
- Defined as the n-th root of the product of n return relatives minus one or $G = \left( (1 + TR_1)(1 + TR_2)...(1 + TR_n) \right)^{1/n} - 1$
- Difference between Geometric mean and Arithmetic mean depends on the variability of returns, $s$

\[(1 + G)^2 = (1 + X)^2 - s^2\]

### Adjusting Returns for Inflation
- Returns measures are not adjusted for inflation
  - Purchasing power of investment may change over time
  - Consumer Price Index (CPI) is possible measure of inflation

\[\text{TR}_A = \frac{(1 + TR)}{(1 + CPI)} - 1\]

### Measuring Risk
- Risk is the chance that the actual outcome is different than the expected outcome
- Standard Deviation measures the deviation of returns from the mean

\[s = \left( \frac{\sum (X - \bar{X})^2}{n - 1} \right)^{1/2}\]

### Risk Premiums
- Premium is additional return earned or expected for additional risk
  - Calculated for any two asset classes
- Equity risk premium is the difference between stock and risk-free returns
- Bond horizon premium is the difference between long- and short-term government securities

### The Risk-Return Record
- Since 1925, cumulative wealth indexes show stock returns dominate bond returns
  - Stock standard deviations also exceed bond standard deviations
- Annual geometric mean return for the S&P 500 is 10.3% with standard deviation of 20.5%
Expected Return and Risk

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Investment Decisions

• Involve uncertainty
• Focus on expected returns
  – Estimates of future returns needed to consider and manage risk
• Goal is to reduce risk without affecting returns
  – Accomplished by building a portfolio
  – Diversification is key

Dealing With Uncertainty

• Risk that an expected return will not be realized
• Investors must think about return distributions, not just a single return
• Probabilities weight outcomes
  – Should be assigned to each possible outcome to create a distribution
  – Can be discrete or continuous

Calculating Expected Return

• Expected value
  – The single most likely outcome from a particular probability distribution
  – The weighted average of all possible return outcomes
  – Referred to as an ex ante or expected return
  \[ E(R) = \sum_{i=1}^{m} R_i p_i \]

Calculating Risk

• Variance and standard deviation used to quantify and measure risk
  – Measures the spread in the probability distribution
  – Variance of returns: \[ s^2 = \sum (R_i - E(R))^2 p_i \]
  – Standard deviation of returns:
    – \[ s = (s^2)^{1/2} \]
  – Ex ante rather than ex post s relevant
Portfolio Expected Return

- Weighted average of the individual security expected returns
  - Each portfolio asset has a weight, \( w \), which represents the percent of the total portfolio value

\[
E(R_p) = \sum_{i=1}^{n} w_i E(R_i)
\]

Portfolio Risk

- Portfolio risk not simply the sum of individual security risks
- Emphasis on the risk of the entire portfolio and not on risk of individual securities in the portfolio
- Individual stocks are risky only if they add risk to the total portfolio

\[
\sigma_p^2 \neq \sum_{i=1}^{n} w_i \sigma_i^2
\]

Risk Reduction in Portfolios

- Assume all risk sources for a portfolio of securities are independent
- The larger the number of securities the smaller the exposure to any particular risk
  - “Insurance principle”
- Only issue is how many securities to hold

Portfolio Risk and Diversification

- Random diversification
  - Diversifying without looking at relevant investment characteristics
  - Marginal risk reduction gets smaller and smaller as more securities are added
- A large number of securities is not required for significant risk reduction
- International diversification benefits

Risk Reduction in Portfolios

- Measured by the variance or standard deviation of the portfolio’s return
  - Portfolio risk is not a weighted average of the risk of the individual securities in the portfolio

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Markowitz Diversification

• Non-random diversification
  – Active measurement and management of portfolio risk
  – Investigate relationships between portfolio securities before making a decision to invest
  – Takes advantage of expected return and risk for individual securities and how security returns move together

Measuring Comovements in Security Returns

• Needed to calculate risk of a portfolio:
  – Weighted individual security risks
    • Calculated by a weighted variance using the proportion of funds in each security
    • For security i: \(w_i \times \sigma_i^2\)
  – Weighted comovements between returns
    • Return covariances are weighted using the proportion of funds in each security
    • For securities i, j: \(2w_i w_j \times \sigma_{ij}\)

Correlation Coefficient

• Statistical measure of association
  • \(\rho_{mn}\) = correlation coefficient between securities m and n
    – \(\rho_{mn} = +1.0\) = perfect positive correlation
    – \(\rho_{mn} = -1.0\) = perfect negative (inverse) correlation
    – \(\rho_{mn} = 0.0\) = zero correlation

Correlation Coefficient

• When does diversification pay?
  – With perfectly positive correlated securities?
    • Risk is a weighted average, therefore there is no risk reduction
  – With zero correlation correlation securities?
  – With perfectly negative correlated securities?

Covariance

• Absolute measure of association
  – Not limited to values between -1 and +1
  – Sign interpreted the same as correlation
  – Correlation coefficient and covariance are related by the following equations:

\[
\sigma_{AB} = \sum_{i=1}^{n} (R_{A,i} - \mu_A)(R_{B,i} - \mu_B)\rho_{i} \\
\rho_{AB} = \sigma_{AB} / \sigma_A \sigma_B
\]

Calculating Portfolio Risk

• Encompasses three factors
  – Variance (risk) of each security
  – Covariance between each pair of securities
  – Portfolio weights for each security
• Goal: select weights to determine the minimum variance combination for a given level of expected return
Calculating Portfolio Risk

- Generalizations
  - the smaller the positive correlation between securities, the better
  - Covariance calculations grow quickly
    - \( n(n-1) \) for \( n \) securities
  - As the number of securities increases:
    - The importance of covariance relationships increases
    - The importance of each individual security’s risk decreases

Simplifying Markowitz Calculations

- Markowitz full-covariance model
  - Requires a covariance between the returns of all securities in order to calculate portfolio variance
    - \( n(n-1)/2 \) set of covariances for \( n \) securities
  - Markowitz suggests using an index to which all securities are related to simplify

An Efficient Portfolio

- Smallest portfolio risk for a given level of expected return
- Largest expected return for a given level of portfolio risk
- From the set of all possible portfolios
  - Only locate and analyze the subset known as the efficient set
    - Lowest risk for given level of return

Efficient Portfolios

- Efficient frontier or Efficient set
  - (curved line from A to B)
- Global minimum variance portfolio
  - (represented by point A)

An Efficient Portfolio

- All other portfolios in attainable set are dominated by efficient set
- Global minimum variance portfolio
  - Smallest risk of the efficient set of portfolios
- Efficient set
  - Part of the efficient frontier with greater risk than the global minimum variance portfolio

Efficient Portfolios

- Efficient frontier or Efficient set
- Global minimum variance portfolio

Nonsystematic Risk

- Variability of a security’s total return not related to general market variability
  - Diversification decreases this risk
- The relevant risk of an individual stock is its contribution to the riskiness of a well-diversified portfolio
  - Portfolios rather than individual assets most important
Portfolio Risk and Diversification

- Total risk
  - Systematic Risk
  - Diversifiable Risk

Capital Asset Pricing Model

- Focus on the equilibrium relationship between the risk and expected return on risky assets
- Builds on Markowitz portfolio theory
- Each investor is assumed to diversify his or her portfolio according to the Markowitz model

Security Market Line

- A security’s contribution to the risk of the market portfolio is based on beta
- Equation for expected return for an individual stock

\[ E(R_i) = RF + \beta_i [E(R_M) - RF] \]

Security Market Line

- Beta = 1.0 implies as risky as market
- Securities A and B are more risky than the market
  - Beta > 1.0
- Security C is less risky than the market
  - Beta < 1.0

Security Market Line

- Beta measures systematic risk
  - Measures relative risk compared to the market portfolio of all stocks
  - Volatility different than market
- All securities should lie on the SML
  - The expected return on the security should be only that return needed to compensate for systematic risk